Analysis of Two-Tone, Third-Order Distortion in Cascaded Two-Ports

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Abstract —We consider the third-order intermodulation distortion (IMD) of a system composed of a number of cascaded two-port networks. The two-tone, third-order intercept point (IP) is highly dependent on the phase angles of the IMD signals, which are usually unknown to the system designer. Consequently, worst-case design strategies are normally used in these situations.

In this paper, we develop general bound formulas for the intercept point that include the effects of mismatches between component networks. We also obtain expressions for the expected value and variance of the intercept point of two cascaded two-port networks. A comparison of these results with measurements indicates that worst-case design strategies are overly conservative in many situations.

I. Introduction

When two signals at closely spaced frequencies, f_1 and f_2, are applied at the input of a nonlinear device, the output contains intermodulation products. Among these, the third-order product signals at the frequencies $2f_1 - f_2$ and $2f_2 - f_1$ are of concern because they are spaced close to the fundamental and cannot be easily eliminated by filtering. A relative comparison of the power levels of these third-order products to the fundamental power level is indicated by the two-tone, third-order intercept point (IP), which is the hypothetical point where the linear and third-order power responses intersect [1], [2].

When two-ports are connected in cascade, an accurate determination of the IP of the overall system requires that the phase angles of the intermodulation spurs be known. As phase measurements of the intermodulation spurs are difficult, an approximate expression for the IP given by Wilson [3] is often used. This expression assumes the worst-case situation, in which all the intermodulation spurs add in phase. In addition, effects due to mismatches between the two-ports are neglected.

In Section II of this paper, we develop exact expressions for the intermodulation products generated by cascaded two-ports and we obtain formulas for the maximum and minimum bounds of system IP. System designers should find these bound formulas useful in estimating the range of IP values that will be realized in practical design situations. We also analyze the IP of cascaded two-ports from a statistical viewpoint in Section III. We obtain expressions for the expected mean value and the variance of the IP for two cascaded two-ports when the phases of the spur signals are assumed to be uniformly distributed. These results are of interest to system designers, who are concerned with producing large numbers of cascaded systems.

The bounds and statistics of the IP that are derived in this paper are compared with experimental measurements in Section IV. Our example shows that a high percentage of measurements fall within the variance about the expected IP value. Consequently, expressions that provide a worst-case estimate of the system IP may be too conservative for many production applications.

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II. Theory

The (ideal) fundamental and third-order responses for a single two-port are shown in Fig. 1. Extrapolations from the linear regions of these responses intersect at the intercept point X. The output power level of X in dBm is expressed as [3]

$$X = \frac{R}{2} + P_o \qquad \text{(dBm)} \tag{1}$$

where P_o is the fundamental output power level (in dBm), R is the relative suppression given by,

$$R = P_o - D \qquad \text{(dB)} \tag{2}$$

and D is the output power (in dBm) of the spurious signal. The power relationships (in mW) between the above quantities are

$$x = r^{1/2} p_o \qquad \text{(mW)} \tag{3}$$

where

$$r = p_o / d \tag{4}$$

and p_o and d are expressed in mW.

Here, we obtain an expression for the intercept point x_T when a number of two-port networks are cascaded as shown in Fig. 2. We assume that the source attached at plane S and the load attached at plane T are matched (i.e., Γ_0 and $\Gamma_{2n+1} = 0$). The reflection coefficients $\Gamma_1, \Gamma_2, \cdots$ and Γ_{2n} include reflections from all networks beyond their reference planes. For example, Γ_3 accounts for reflections from networks 2 to n while Γ_4 accounts for reflections

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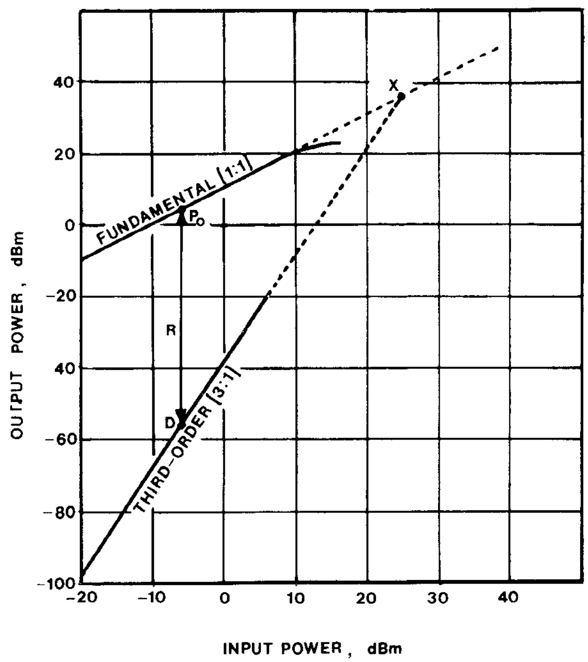

Fig. 1. Fundamental and third-order response of a two-port device showing intercept point.

from networks 1 and 2. We also define the following notation:

S_{T_i} = total forward transmission coefficient of components $i+1, i+2, \cdots, n$ connected in cascade. Evaluation of these coefficients should include mismatches that occur between the various networks, i.e., $S_{T_i} \neq \prod_{j=i}^{n} S_{j+1}$, unless $\Gamma_j = 0$.

b_i = complex power wave corresponding to the intermodulation (IM) product produced by the ith network at a matched output.

The complex power waves b_1, b_2, and b_n are shown in Fig. 2 along with b_T, which is the total third-order distortion signal at the system output. Since the system input and output are matched, $S_{Tn} = 1$ and

$$b_T = b_1 \frac{S_{T1}}{1-\Gamma_2\Gamma_3} + b_2 \frac{S_{T2}}{1-\Gamma_4\Gamma_5} + \cdots + b_r \frac{S_{Tr}}{1-\Gamma_{2r}\Gamma_{2r+1}} + \cdots + b_n. \quad (5)$$

We can express the IMD output signal power d_T (in mW) as

$$d_T = |b_T|^2 = \sum_{i=1}^{n} d_i \cdot g_i + 2 \sum_{\substack{i=2 \\ i>j}}^{n} \sum_{j=1}^{n} \sqrt{d_i \cdot g_i \cdot d_j \cdot g_j} \cdot \cos(\phi_i - \phi_j + \theta_i - \theta_j) \quad (6)$$

where

$$\sqrt{g_i}\, e^{j\theta_i} \triangleq \frac{S_{T_i}}{1-\Gamma_{2i}\Gamma_{2i+1}} = \left| \frac{S_{T_i}}{1-\Gamma_{2i}\Gamma_{2i+1}} \right| e^{j\theta_i} \quad (7)$$

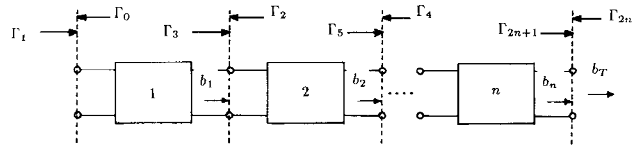

Fig. 2. Cascade connection of n nonlinear two-ports.

and

$$b_i \triangleq \sqrt{d_i}\, e^{j\phi_i}. \quad (8)$$

We note that g_i and θ_i in (7) are expressible in terms of the scattering parameters of the two-ports at the fundamental frequency because the spur frequency is spaced close to the fundamental. These scattering parameters can be individually measured with modern network analyzers and the quantities g_i and θ_i can be calculated. The IMD power levels d_i given in (8) can also be measured but the phases ϕ_i, which are very important in determining the system output power of the IMD signal, are usually unknown.

We obtain the relative suppression of the distortion product at the system output r_T by dividing d_T by the output power level of the fundamental signal p_o. Without loss in generality, we can equate the output power:

$$p_o = p_i g_i, \qquad i = 1, 2, \cdots, n \quad (9)$$

where p_i is the power incident on a matched load at the output of the ith network and g_i accounts for all mismatches between the ith network and the system output. Consequently, the relative suppression is

$$r_T^{-1} = \frac{d_T}{p_o} = \sum_{i=1}^{n} \frac{d_i \cdot g_i}{p_i \cdot g_i} + 2 \sum_{\substack{i=2 \\ i>j}}^{n} \sum_{j=1}^{n} \left(\frac{d_i \cdot g_i \cdot d_j \cdot g_j}{p_i \cdot g_i \cdot p_j \cdot g_j} \right)^{1/2} \cdot \cos(\phi_i - \phi_j + \theta_i - \theta_j)$$
$$= \sum_{i=1}^{n} (r_i)^{-1} + 2 \sum_{\substack{i=2 \\ i>j}}^{n} \sum_{j=1}^{n} (r_i)^{-1/2} (r_j)^{-1/2} \cdot \cos(\phi_i - \phi_j + \theta_i - \theta_j) \quad (10)$$

where r_i is the relative suppression at the output of the ith network. Multiplying both sides of (10) by p_o^{-2} and using (3) and (9), we get an expression for the intercept point for the overall system:

$$x_T = \left[\sum_{i=1}^{n} \frac{1}{x_i^2 g_i^2} + 2 \sum_{\substack{i=2 \\ i>j}}^{n} \sum_{j=1}^{n} \frac{\cos(\phi_i - \phi_j + \theta_i - \theta_j)}{x_i x_j g_i g_j} \right]^{-1/2} \quad (11)$$

where x_i is the intercept point of the ith component. The above expression for the IP is exact and can be easily included in computer-aided design (CAD) software. However, phase information about the spur signals is seldom available so we must estimate x_T by making assumptions about ϕ_i.

Maximum and Minimum Bounds on X_T

The minimum value of x_T occurs when the argument of the cosine term in (11) is zero. In that case

$$x_T^{\min} = \left(\sum_{i=1}^{n} \frac{1}{x_i g_i} \right)^{-1}. \tag{12}$$

If the networks are matched to each other, (12) agrees with the result given in [3].

The maximum value of x_T occurs when the argument of the cosine term in (11) is π or some odd multiple thereof:

$$x_T^{\max} = \left(\sum_{i=1}^{n} \frac{1}{x_i^2 g_i^2} - 2 \sum_{\substack{i=2 \\ i>j}}^{n} \sum_{j=1}^{n} \frac{1}{x_i x_j g_i g_j} \right)^{-1/2}. \tag{13}$$

This result represents the best possible situation where the intercept point for a cascaded system reaches its highest possible value. In general, however, the IP can fall anywhere between the bounds given by (12) and (13).

III. Statistical Analysis of the Intercept Point

We have seen that the phase angle relationships between the intermodulation spur signals and the scattering parameters determine the magnitude of the overall IP of a cascaded system. Depending on the realization of these phase angles, the IP of a cascaded system can range between $x_T^{\min}$ and $x_T^{\max}$. In the past, a safe strategy has been to assume the worst case value $x_T^{\min}$ when designing a system. Although conservative, this approach guarantees that the intermodulation products always fall below a specified limit.

The above design approach may not be optimum when designing systems for production, however. We may be willing to produce a number of systems that fail the overall IP design goal if 1) we can identify these faulty systems by testing before shipment and 2) the production yield is reasonably high. To adopt this alternative approach, we consider the problem from a stochastic viewpoint. We assume that the phase angle realizations of the spur signals are random variables and we evaluate the expected value $E(x_T)$ and the variance of the intercept point.

For the special case of two cascaded two-port networks, $n = 2$ and (11) simplifies to

$$x_T = \frac{A}{\sqrt{B + \cos(\phi_1 - \phi_2 + \theta_1)}} \tag{14}$$

where

$$A = \frac{x_1 x_2 g_1}{2} \quad \text{and} \quad B = \frac{1}{2}\left(\frac{x_2}{x_1 g_1} + \frac{x_1 g_1}{x_2} \right). \tag{15}$$

The minimum and maximum values of x_T are $A/\sqrt{B+1}$ and $A/\sqrt{B-1}$, respectively. Assuming ϕ_1 and ϕ_2 as independent random variables distributed uniformly between 0 and 2π, the probability density function (pdf) $f_Y(y)$ of the random variable $y = \cos(\phi_1 - \phi_2 + \theta_1)$ is obtained [4] as

$$f_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}, \qquad |y| < 1. \tag{16}$$

The expected value of x_T is therefore

$$E(x_T) = \int_{-1}^{1} \frac{A}{\sqrt{B+y}} \cdot \frac{1}{\pi\sqrt{1-y^2}} \cdot dy = \frac{2A}{\pi\sqrt{B+1}} K\left(\sqrt{\frac{2}{B+1}} \right) \tag{17}$$

where $B > 1$ and $K(\cdot)$ is the complete elliptic integral of the first kind. The mean square value of x_T can also be obtained as

$$E(x_T^2) = \int_{-1}^{1} \frac{A^2}{(B+y)} \frac{1}{\pi\sqrt{1-y^2}} \cdot dy = \frac{2A^2}{\pi\sqrt{B^2-1}} \left[\tan^{-1} \frac{B+1}{\sqrt{B^2-1}} - \tan^{-1} \frac{1-B}{\sqrt{B^2-1}} \right]. \tag{18}$$

When the number of nonlinear elements is greater than two, determination of the expected value and variance of x_T becomes more involved. In those cases where three or more subsystems contribute to the overall intercept point value, a possible approach is to perform a Monte Carlo simulation of the system.

In many cases, however, the generation of intermodulation products can be attributed to one or two of the cascaded two-ports, so the above analysis is a valid approximation. Hence, the above analysis will be valid if the intermodulation distortion signals are primarily generated by two of the cascaded networks.

IV. An Example and Conclusions

Equation (11) allows us to compute the intercept point of a system of cascaded two-port networks if we know the scattering coefficients and the intercept point of each network and the phases of each IMD spur. Since the spur phases will usually be unknown, designers must be satisfied with knowing the bounds and the first-order statistics of the intercept point. Although these calculations are straightforward in principle, they are tedious and are best done with a computer. In the following example, we use scattering-parameter-based CAD software that was written specifically for the purpose of analyzing general microwave systems [5]. Called CAAMS (Computer-Aided Analysis of Microwave Systems), this software package was developed by the University of Massachusetts for Sanders Associates.

For illustration purposes, we consider the simple example of the two cascaded Watkins-Johnson amplifiers shown in Fig. 3. An attenuator was inserted between the amplifiers to control the operating level of the second amplifier. The small-signal scattering parameters of the component networks were measured with a vector network analyzer in the 400–1500 MHz frequency range, and catalog values of

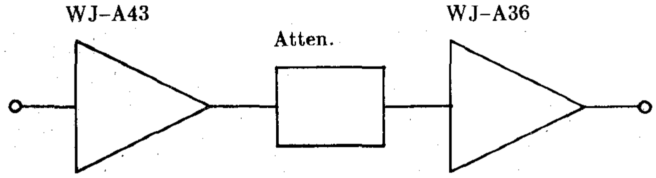

Fig. 3. Block diagram of example cascaded system.

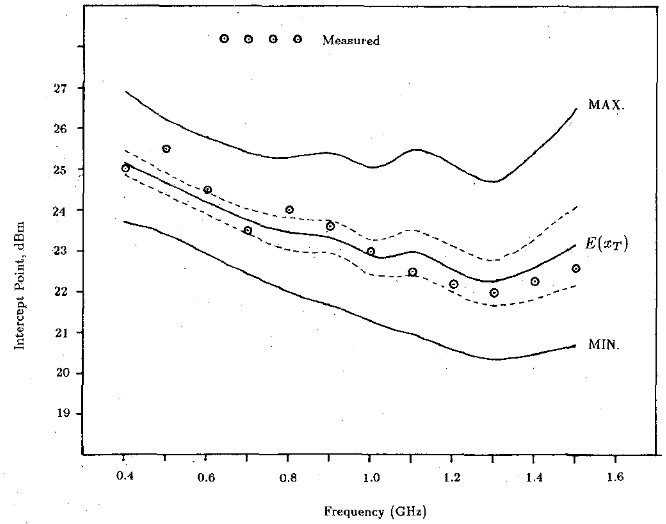

Fig. 4. Measured values of the intercept point for the example system shown in Fig. 3. Also shown are the calculated bounds given by (12) and (13) as well as the expected value and standard deviation of x_T.

the intercept points provided by the manufacturer were used in our analysis of the intercept point for the overall system.

Computed values of the minimum, maximum, expected value, and standard deviation of the overall system intercept point are shown in Fig. 4 as a function of frequency. Measured values of the system intercept point are also shown as data points. The system IP values were obtained by measuring the system output power levels at the fundamental and the third-order IMD frequencies as a function of the power levels of the input tones.

Frequency variations of the computed bounds, mean value, and standard deviation of the IP are due primarily to the passband and matching characteristics of the amplifiers used in this example. Although most of the measured intercept points fall within a standard deviation of the mean value, we remind the reader that the measurements can fall anywhere between the two limiting bounds. We can expect that measurements of a large ensemble of similar systems would average to the expected value shown, but the IP values for a single system will range between $X_T^{\min}$ and $X_T^{\max}$.

Our example system includes two amplifiers that contribute IMD signals and a passive linear attenuator that does not. Although passive linear networks have intercept points that are infinite, they still influence the overall system IP through their transmission and reflection properties. In fact, it is possible to tune IMD signal levels generated by a cascaded system with a linear network element.

Our analysis of two-tone, third-order distortion shows that CAD techniques can effectively estimate the effects of IMD signals in cascaded two-ports. In addition to calculating the maximum and minimum bounds of the intercept point, one can evaluate its expected value and variance. The latter quantities are useful in getting a handle on the performance statistics that a large number of cascaded circuits will achieve in production.

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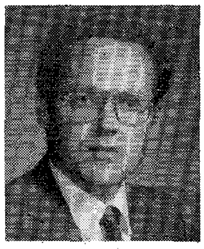

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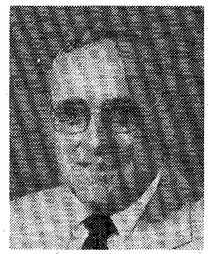

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